



Government of **Western Australia**  
School Curriculum and Standards Authority



## Western Australian Certificate of Education Examination, 2016

### Question/Answer Booklet

# MATHEMATICS SPECIALIST

## Section Two: Calculator-assumed

Please place your student identification label in this box

Student Number: In figures

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In words

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### Time allowed for this section

Reading time before commencing work: ten minutes  
Working time for this section: one hundred minutes

### Materials required/recommended for this section

#### *To be provided by the supervisor*

This Question/Answer Booklet  
Formula Sheet (retained from Section One)

Number of additional  
answer booklets used  
(if applicable):

#### *To be provided by the candidate*

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,  
correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper,  
and up to three calculators approved for use in the WACE examinations

### Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.



**Structure of this paper**

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	54	35
Section Two: Calculator-assumed	14	14	100	102	65
<b>Total</b>					100

**Instructions to candidates**

- The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2016*. Sitting this examination implies that you agree to abide by these rules.
- Write your answers in this Question/Answer Booklet.
- You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.
- Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
  - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
  - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.
- Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
- It is recommended that you **do not use pencil**, except in diagrams.
- The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

**Section Two: Calculator-assumed**

**65% (102 Marks)**

This section has **14** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 100 minutes.

**Question 8**

**(6 marks)**

The system of linear equations given below can be reduced in three stages to a form where it can be solved easily.

$$\begin{array}{rcl} x + y + z = 4 & \dots R_1 \\ 2x + 3y + z = 8 & \dots R_2 \\ 3x + (3 - p)y + 2z = 13 - p^2 & \dots R_3 \end{array}$$

- (a) Two of the stages are given below.

In the space provided at the side of each stage, write the operation(s) that have been performed in terms of the rows of the previous system.

**(2 marks)**

$$\begin{array}{rcl} x + y + z = 4 & \dots R_1 \\ 2x + 3y + z = 8 & \dots R_2 \\ py + z = p^2 - 1 & \dots R_3 \end{array}$$

$$\begin{array}{rcl} x + y + z = 4 & \dots R_1 \\ y - z = 0 & \dots R_2 \\ py + z = p^2 - 1 & \dots R_3 \end{array}$$

- (b) Perform one further row operation so that the coefficient of  $z$  in  $R_3$  is 0.

**(1 mark)**

**Question 8** (continued)

- (c) For each of  $p = 1$  and  $p = -1$  indicate how many solutions there are to the system of equations. If there is a unique solution, give that solution. If there is an infinite number of solutions, give the resulting solution when  $z = -1$ . (3 marks)

Question 9

(8 marks)

The standard deviation of the durability of Performance Racing tyres is 410 kilometres.

Racing experts plan to estimate  $\mu$ , the mean lifetime of these tyres, using the mean lifetime of a random sample of the tyres.

- (a) The experts would like to be 95% confident that the mean lifetime of tyres in the sample is within 50 kilometres of  $\mu$ . How large a sample should they take? (3 marks)

- (b) Suppose that a random sample of 80 tyres is taken, and the mean lifetime of these tyres is 1245 kilometres. Based on this sample, determine a 90% confidence interval for  $\mu$ . (3 marks)

- (c) The manufacturer claims that the mean lifetime of Performance Racing tyres is at least 1250 kilometres. Does the sample in part (b) provide a strong reason to doubt this claim? Justify your answer. (2 marks)

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Question 10

(7 marks)

- (a) Let  $A$  be a point **not** on the line  $L$  that passes through the points  $B$  and  $C$ .

Given  $|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}| |\mathbf{b}| \sin \theta$  show that the distance  $d$  from the point  $A$  to the line  $L$  is

$$d = \frac{|\mathbf{a} \times \mathbf{b}|}{|\mathbf{a}|}$$

where  $\mathbf{a} = \overrightarrow{BC}$  and  $\mathbf{b} = \overrightarrow{BA}$ .

(3 marks)

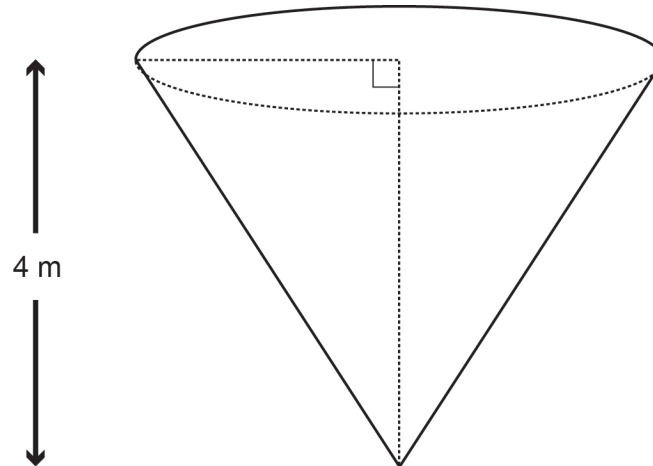
- (b) Use the formula in part (a) to find the distance from the point  $A(1, 1, 1)$  to the line through  $B(0, 6, 8)$  and  $C(-1, 4, 7)$ . (4 marks)

Question 11

(9 marks)

- (a) The ethanol produced by a chemical factory is poured into a 4 m high conical container, with an upper diameter of 4 m, at a constant rate of  $3 \text{ m}^3$  per minute. At what rate is the ethanol level rising in the container when the depth of the ethanol is exactly 2.5 m?

(5 marks)



**Question 11** (continued)

- (b) Determine the equation of the graph that has the following characteristics:
- at each point  $(x, y)$  on the graph of the function, the gradient of the tangent at that point is given by  $\frac{-x}{2y}$
  - the graph passes through the point  $(2, 1)$ . (4 marks)



Question 12

(7 marks)

The equation of a sphere is given by  $4x^2 + 4y^2 + 4z^2 + 16y - 24x + 32z = 612$ .

- (a) Determine the vector equation of the sphere.

(3 marks)

- (b) Determine the position vector(s) of the points of intersection between the sphere and the line  $\mathbf{r} = -3\mathbf{i} + 5\mathbf{j} + \mathbf{k} + \lambda(-2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$ .

(4 marks)

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## Question 13

(8 marks)

- (a) A research scientist wished to estimate, with a 95% confidence interval, the mean amount of moisture absorbed through the skin of a particular species of large animal in a laboratory experiment. The scientist believed that the values were normally distributed, and from past experience, felt that the population variance was 4 grams.

A random sample of 25 of this particular species yielded a mean rate of 16.5 grams of moisture being absorbed. Determine the 95% confidence interval for this experiment and explain your findings. (4 marks)

- (b) The same scientist decided that she needed further evidence of the true size of the population mean of moisture absorbed for this particular species. She wished to be within 0.7 grams of the true mean with 99% confidence. What size sample would the scientist need? (4 marks)

Question 14

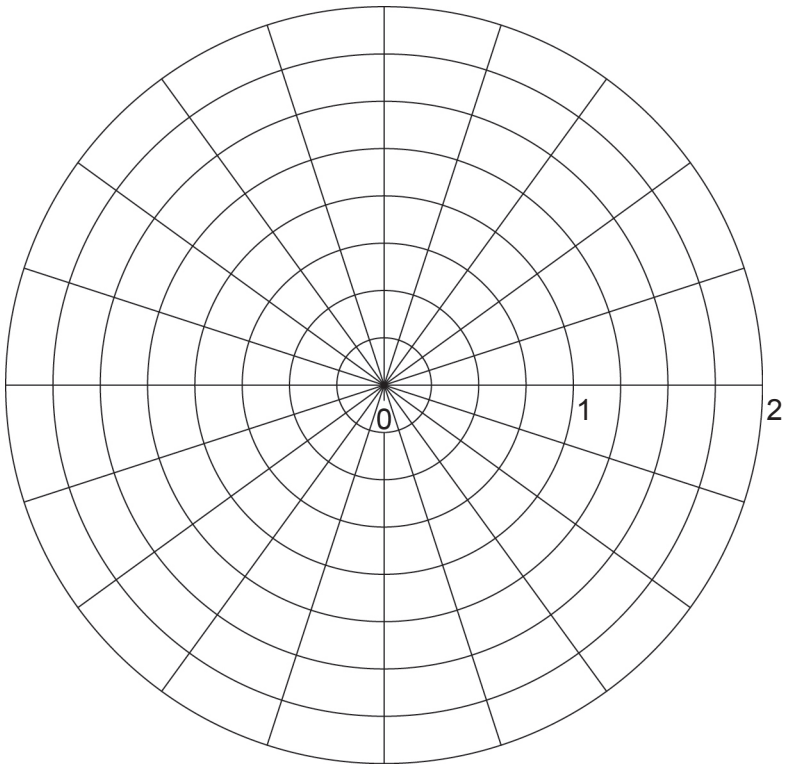
(10 marks)

- (a) Determine all of the roots of the equation  $z^6 = \sqrt{3} + i$ , expressing them in polar form  $r\text{cis } \theta$  where  $r \geq 0$  and  $-\pi < \theta \leq \pi$ . (5 marks)

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Question 14 (continued)

- (b) Plot the roots found in part (a) on the diagram below.
- (3 marks)



- (c) The roots form the vertices of a hexagon. Determine the exact value for the perimeter of the hexagon.
- (2 marks)

Question 15

(6 marks)

An engine piston undergoes simple harmonic motion which can be described by the differential equation  $\frac{d^2x}{dt^2} = -9x$ , where  $x$  metres is the displacement of the piston from its mean position at  $t$  seconds.

- (a) Determine the period of the motion. (1 mark)

- (b) If the maximum speed of the piston is 5 m/s, calculate the amplitude of the motion. (2 marks)

- (c) The amplitude and period of the motion are now changed, but the piston still undergoes simple harmonic motion. These new readings are taken:  
when  $x = 1$  m, speed =  $\sqrt{60}$  m/s;      when  $x = 3$  m, speed =  $\sqrt{28}$  m/s.

Find the new exact values for the:

- (i) period  
(ii) amplitude. (3 marks)

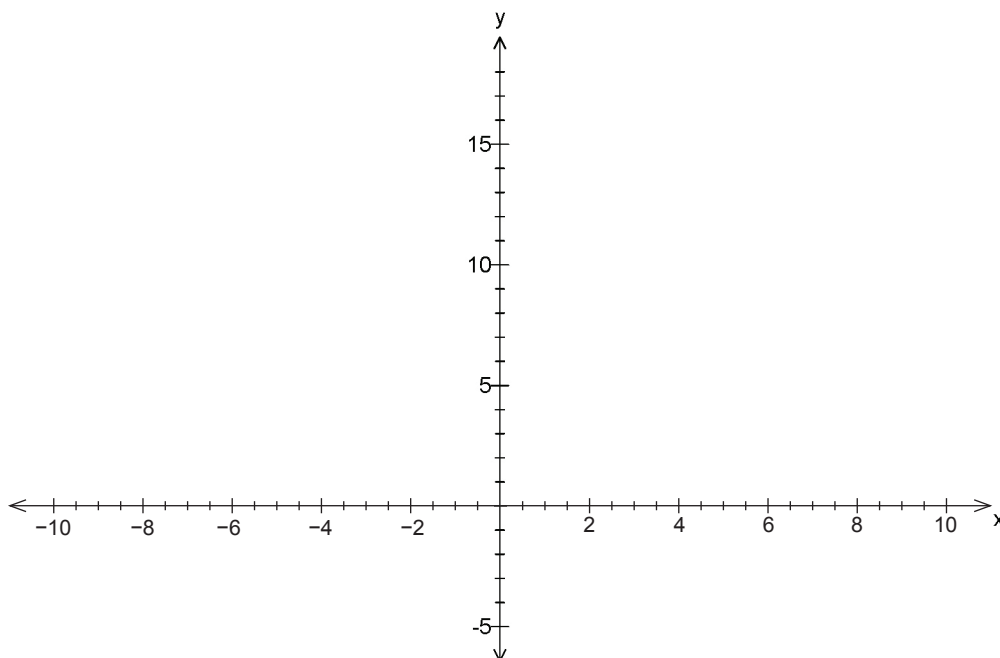
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Question 16

(5 marks)

- (a) Sketch the graph of  $y = |3x + 6|$  on the grid below.

(2 marks)



- (b) Use the graph to determine the values of the real constants  $p$ ,  $q$  and  $r$  if the equation

$$|3x + 6| = p|x + q| + r$$

is satisfied for all  $x \in [-2, 3]$  but no other real values.

(3 marks)

**Question 17**

**(6 marks)**

A designer of applications ('apps') for a major manufacturer of tablet computers is trying to establish the best price to charge for a new app that has been developed. On the basis of previous sales of similar apps, it has been established that the demand function for the product is modelled by

$$p = \frac{3}{0.000\,001x^3 + 0.01x + 1}$$

where  $p$  is measured in dollars and  $x$  is measured in hundreds of units.

- (a) Find the rate of change of the demand  $x$  with respect to the price  $p$ . (3 marks)

- (b) Find the rate of change of the demand  $x$  with respect to the price  $p$  when  $x = 100$  and explain clearly the significance of your answer with respect to the designer. (3 marks)

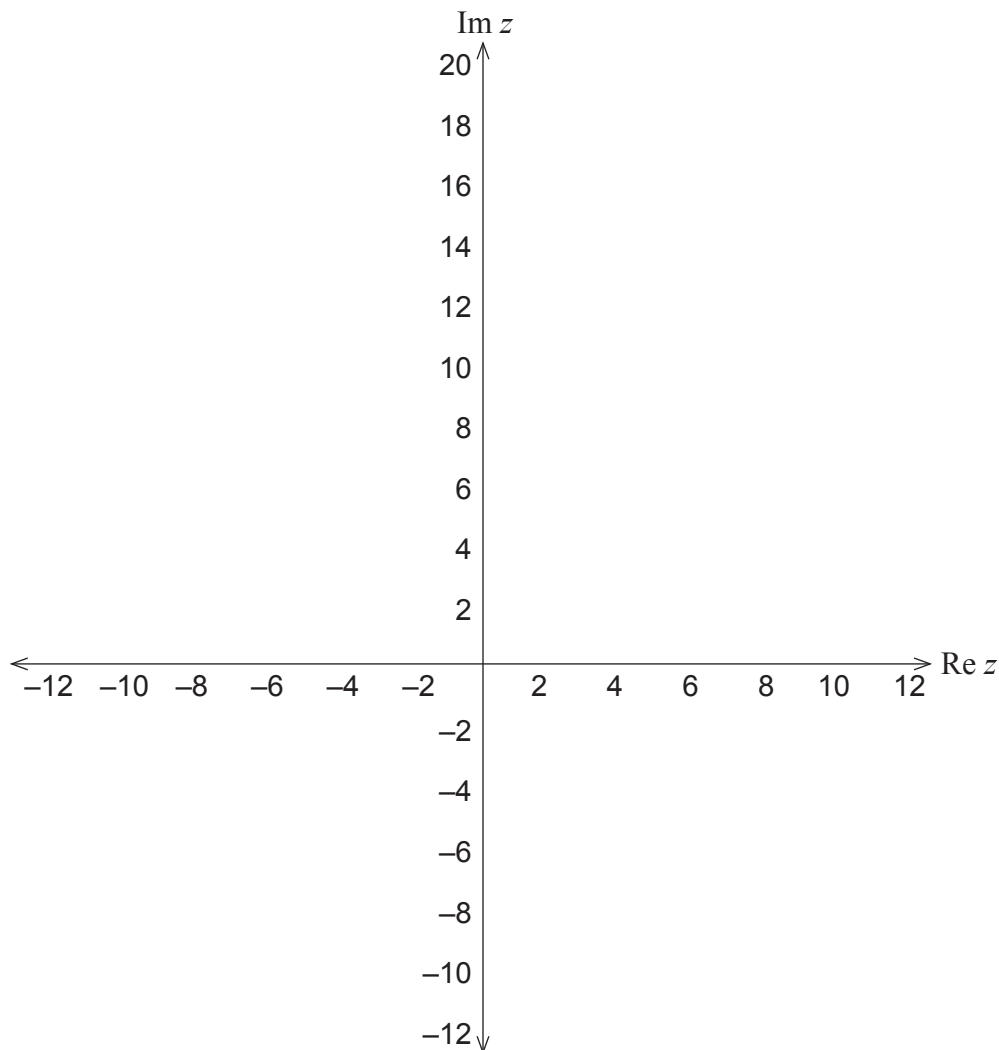
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Question 18

(7 marks)

- (a) On the Argand diagram below, sketch the inequality defined by  $\text{Im } z \leq -2 \text{Re } z + 17$ .

(2 marks)



- (b) Show that the point  $(3, i)$  satisfies the inequality from part (a).

(1 mark)

- (c) The set of points in the complex plane that satisfy  $|z - 3 - i| = |z - a - bi|$ , where  $a$  and  $b$  are certain real constants, can alternatively be defined by the property that they lie on the line  $\text{Im } z = -2 \text{Re } z + 17$ . Determine the values of  $a$  and  $b$ .

(4 marks)



Question 19

(10 marks)

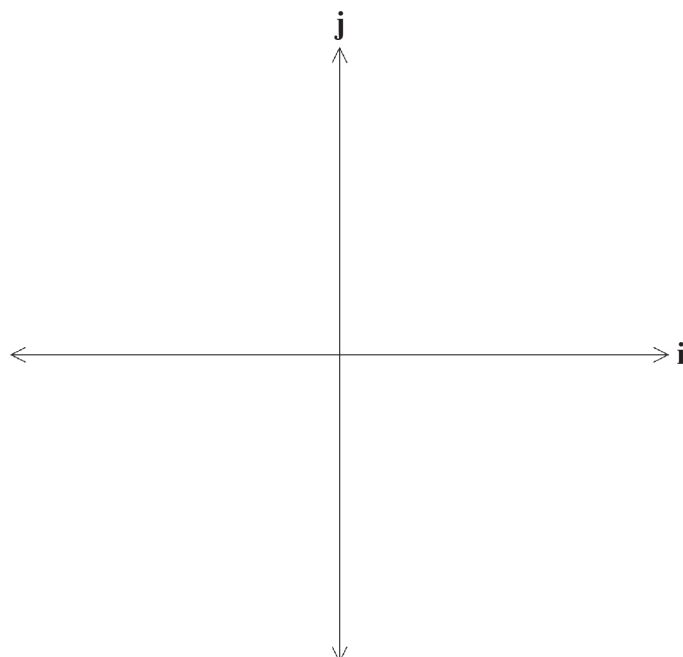
The velocity vector of a moving object at time  $t$  seconds is

$$\mathbf{v}(t) = -5 \sin\left(\frac{t}{2}\right)\mathbf{i} + 4 \cos\left(\frac{t}{2}\right)\mathbf{j}.$$

- (a) Determine the position vector  $\mathbf{r}(t)$  of the object, given that initially,  $\mathbf{r} = 10\mathbf{i}$ . (2 marks)

- (b) Show that the acceleration vector is always parallel to the position vector. (2 marks)

- (c) Sketch the path taken by the object and indicate the direction of travel on the axes below. (2 marks)



See next page

**Question 19** (continued)

- (d) Given that  $\int_0^{4\pi} v(t) dt = 0$ , explain what this means in terms of the path taken by the object. (2 marks)

- (e) Evaluate  $\int_0^{4\pi} |v(t)| dt$  and explain what this means in terms of the path taken by the object. (2 marks)

**Question 20**

**(7 marks)**

The present population,  $P$ , of snakes on a small island is 154. Due to favourable breeding conditions and the availability of a steady food supply, the growth rate of the population in the future is expected to be given by

$$\frac{dP}{dt} = 0.16P\left(1 - \frac{P}{500}\right), \text{ where } t \text{ is the time, in months, from today.}$$

- (a) Express  $P$  as a function of  $t$ . [Hint: Use partial fractions] (5 marks)

- (b) Calculate the approximate numbers of snakes on the island after two years. (1 mark)

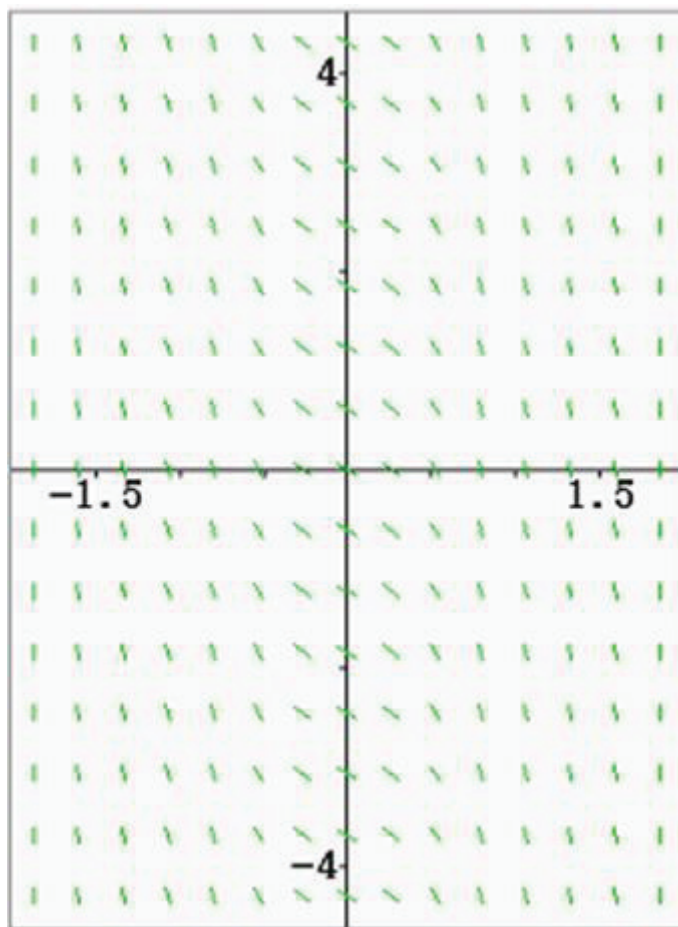
- (c) What is the limiting population size? (1 mark)

**See next page**

Question 21

(6 marks)

A first-order differential equation has a slope field as shown in the diagram below.



- (a) Use the scale shown to determine a general differential equation that would result in this slope field. (2 marks)
- (b) Give **two** reasons for your answer in part (a). (2 marks)
- (c) Determine a possible general equation for  $y$ . (2 marks)

End of questions

**Additional working space**

Question number: \_\_\_\_\_

Additional working space

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